Bandwidth And Sampling Rate In Digitizing Oscilloscopes

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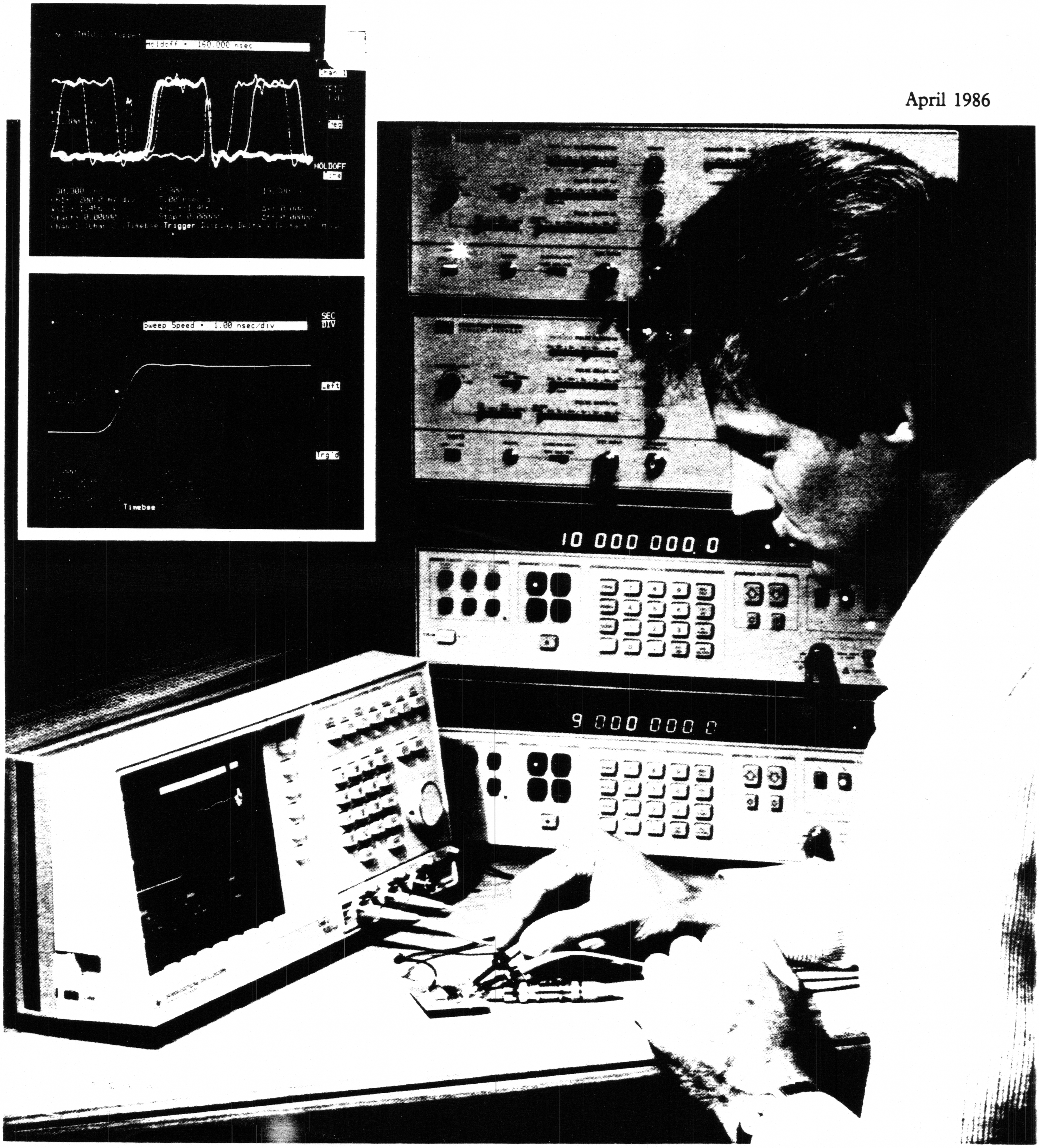

Clearing Up The Relationship Between Bandwidth And Sampling Rate

Bandwidth, along with its time-domain equivalent, rise time, has historically been the principal figure of merit for oscilloscopes. With the increasing popularity of digitizing oscilloscopes, sampling rate, or digitizing rate, has attracted almost equal attention as a figure of merit. The relationship between bandwidth and sampling rate has given rise to a lot of confusion. The purpose of this paper is to examine and clarify this relationship.

The following table illustrates the possibilities for confusion.

MODEL AND MANUFACTURER	REPETITIVE BANDWIDTH	SINGLE-SHOT BANDWIDTH	SAMPLING RATE
HP 54100A/D	1 GHz	Not specified	40 Msamples/second
HP 54200A/D	50 MHz	50 MHz	200 Msamples/second
Tektronix 7D20	70 MHz	Not specified	40 Msamples/second
Gould/Biomation 4500	35 MHz	35 MHz	100 Msamples/second

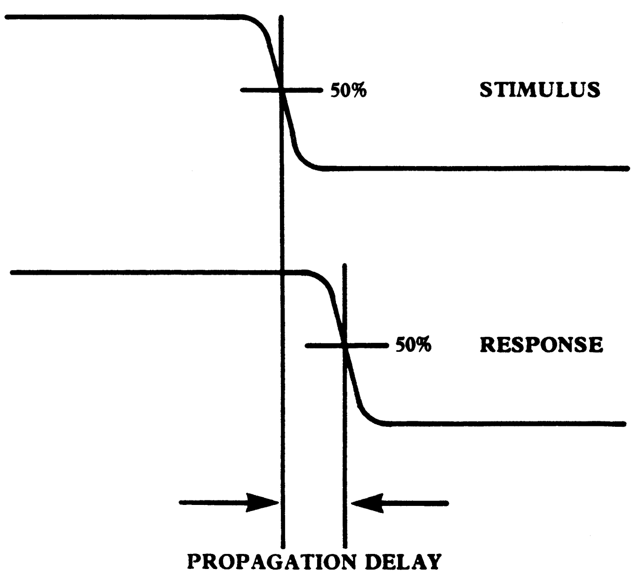

Figure 1 - Propagation delay measurement

As a starting point, let's review why bandwidth and sampling rate are important. Oscilloscopes are not generally used to look at well-behaved, regular signals like sine waves, so who cr whether a sine wave is 3 dB down in amplitude at the specified frequency? What matters are the time and voltage relationships in irregular waveshapes, and how they are affected by bandwidth.

How Bandwidth Affects Measurements

Most oscilloscope measurements are time-interval measurements. Consider a propagation delay measurement (figure 1). The question to be answered by this measurement is, "What is the time interval from the 50% point on the stimulus transition to the 50% point on the response transition?" Note that the time interval is defined in terms of a change in voltage. When the input voltage changes, the oscilloscope cannot exactly reproduce the input signal at every instant in time, due to the transient response of the oscilloscope. In general, the greater the oscilloscope's bandwidth (conversely, the less its rise time), the smaller the errors in measuring time intervals. A good rule of thumb to use is that the oscilloscope's rise time should be less than one-third the time interval to be measured.

Bandwidth also affects amplitude measurements in the time domain. Consider measuring the height of a narrow glitch (figure 2). In a digital circuit, it may be important to know whether the glitch crosses the logic threshold. If the oscilloscope's bandwidth is insufficient, there may be a significant error in measuring the peak height of the glitch (figure 3). In general, the oscilloscope's rise time should be less than one-third the 50% width of the narrowest pulse to be measured.

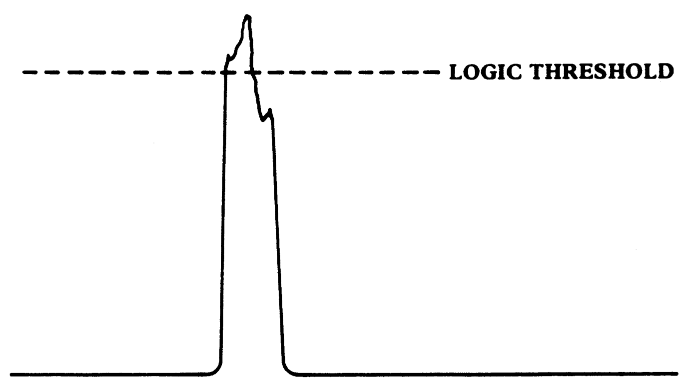

Figure 2 - Measuring the height of a glitch to determine whether it crosses the logic threshold

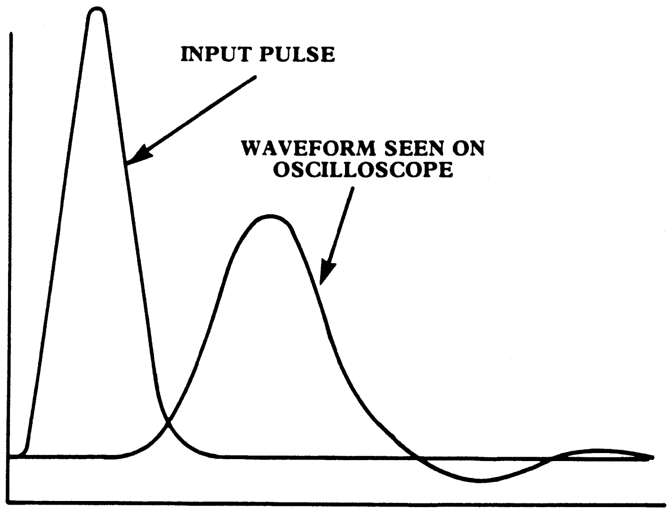

Figure 3 - A narrow pulse applied to an oscilloscope

Introduction To Sampling

In a digitizing oscilloscope, the situation is further complicated because the waveform is quantized into discrete time and voltage samples. To add to the confusion, there are at least three methods of sampling the signal: real-time sampling, sequential sampling, and random repetitive sampling. Each of these sampling methods impacts measurements differently, so a brief description of each is in order.

In real-time sampling, the signal is digitized on the fly, in real time. There is a simple 1:1 correspondence between the samples and the times at which they were taken (figure 4). The advantage of real-time sampling is in single-shot measurements. All the data about the signal is acquired in one acquisition cycle.

In sequential sampling, only one sample of the signal is digitized on each occurrence of the trigger (figure 4). With each successive trigger, the sampling point is delayed further from the trigger point. After many samples are acquired and digitized, the signal is reconstructed in the oscilloscope's digital memory. Sequential sampling requires that the signal be repetitive.

Random repetitive sampling (figure 4) is similar to sequential sampling, except that the signal is constantly sampled and digitized at a rate determined by the oscilloscope's sampling clock. To determine the time relationship of each sample to the trigger, the time between the sample clock and the trigger event is measured whenever a trigger event is detected. Again, as in sequential sampling, the signal data is reconstructed after many samples are acquired, so a repetitive signal is required.

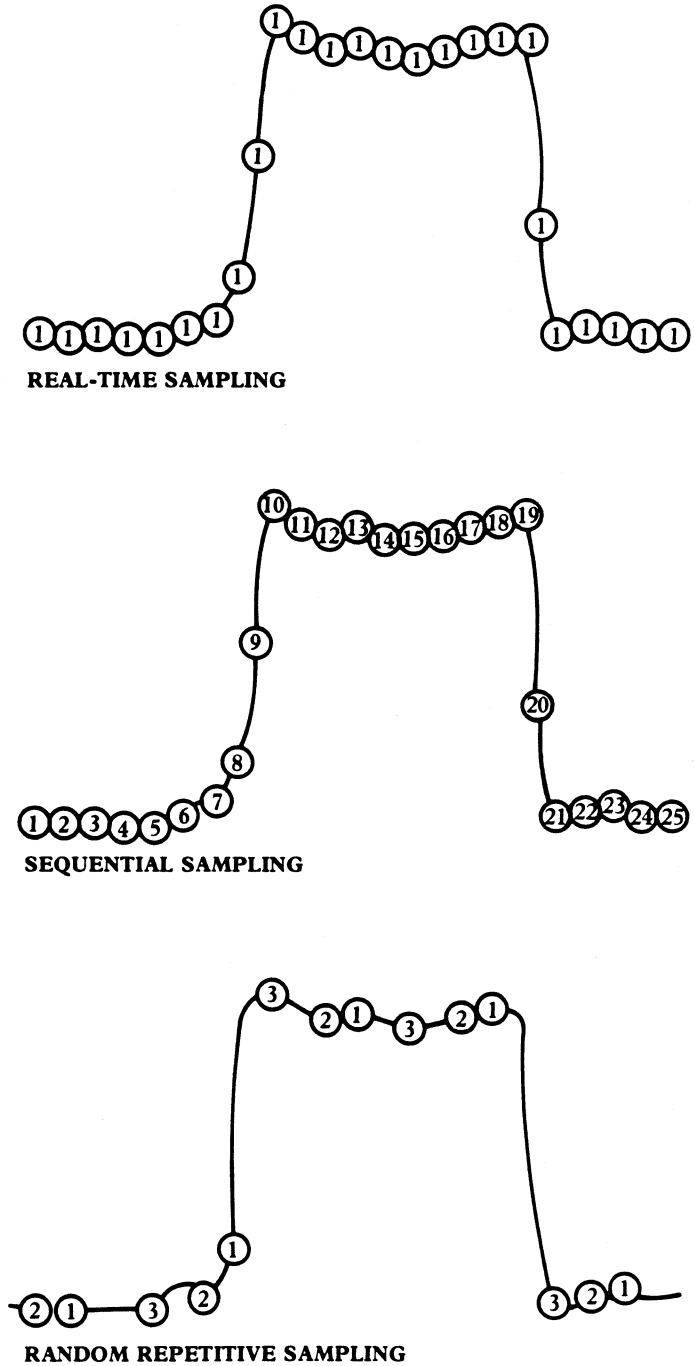

Figure 4 - The number on each sampled point indicates the acquisition cycle on which that point was sampled. Sequential sampling and random repetitive sampling require repetitive signals to reconstruct the digitized signal from samples acquired on successive acquisition cycles. Real-time sampling acquires all the samples on one cycle.

How Sampling Rate Affects Measurements

Let's examine the simplest situation (i.e., real-time sampling) to determine the effect of sampling rate on measurements. We'll then compare that to the effect of bandwidth to illustrate the relationship.

Consider measuring a signal with a relatively fast transition (figure 5). When the samples are not spaced closely enough, we can't know the location of the edge very precisely, nor can we ascertain its shape. The effect is the same as using an analog oscilloscope with insufficient bandwidth. If the oscilloscope has a higher sampling rate as in figure 6, the edge can be located correctly in time and its rise time can be measured.

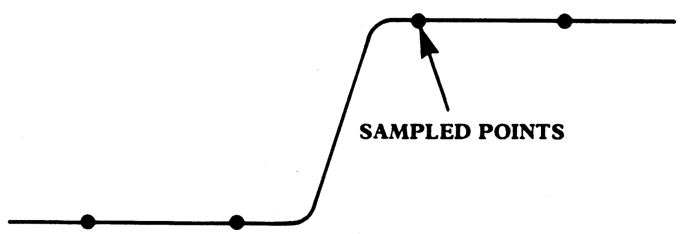

Figure 5 - The edge cannot be located precisely due to undersampling the signal.

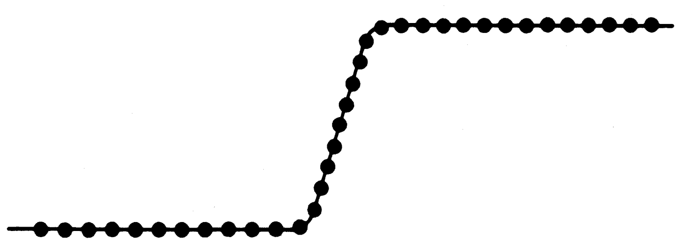

Figure 6 - With a higher sampling rate, the edge can be located with good resolution, and its characteristics may be determined.

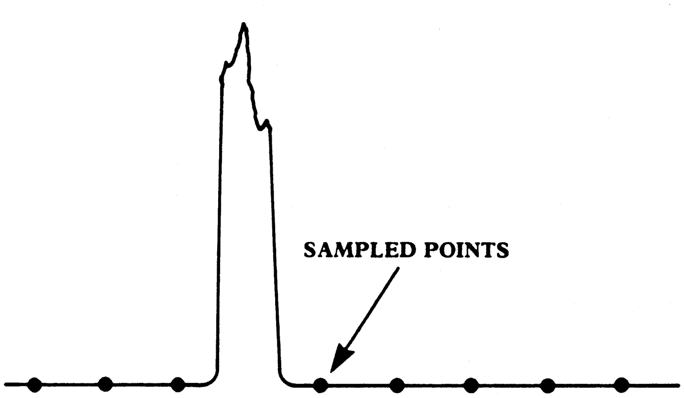

Figure 7 - A glitch may be missed altogether if the sample interval is wider than the glitch.

Going back to the example of a narrow glitch (figure 7), if the sampling rate is insufficient, the glitch may be missed altogether, or its amplitude may be incorrectly represented. Again, the effect is the same as using an oscilloscope with insufficient bandwidth.

Nyquist Versus Bandwidth

Nyquist's theorem states that if a signal is sampled at a frequency 2F, there is no information in the samples about the components of the signal at frequencies above F. The practical implication is that the effect on time-domain measurement uncertainty of a sample rate 2F is equivalent to a band-limit filter with a sharp cutoff at a frequency= F. Note: this should not be extrapolated to read, "The bandwidth of a real-time digitizing oscilloscope is half the sampling rate." The Nyquist limit is an absolute upper limit; there can be no information above the Nyquist limit. Landau (1967, Ref. 4) proved that, "data cannot be transmitted as samples at a rate higher than the Nyquist rate, regardless of the location of the sampling instants, the nature of the set of frequencies which the signals occupy, or the methods of construction." This situation is quite different from the typical Gaussian amplifier response.

Nyquist's theorem comes out of information theory. The error rate or signal-to-noise ratio of a communication channel as a function of information transfer rate is affected in similar ways by bit rate (for digital transmission) and by bandwidth (for analog transmission).

In oscilloscope measurements, the implication of Nyquist's theorem is that we can determine times and voltages on a signal using a real-time digitizing oscilloscope with approximately the same error as would result from using an analog oscilloscope with a bandwidth equal to half the sampling rate in the optimum case.

Thus, in selecting a real-time digitizing oscilloscope, the digitizing rate should be selected to be at least six divided by the smallest rise time or pulse width to be measured.

To illustrate why this is not quite the same as simply stating that the bandwidth is half the sample rate, consider a sine wave with a frequency F, where 2F is the sample rate (figure 8). In this situation, it is obvious from the figure that we can't even be sure there is any signal present, much less what its amplitude is.

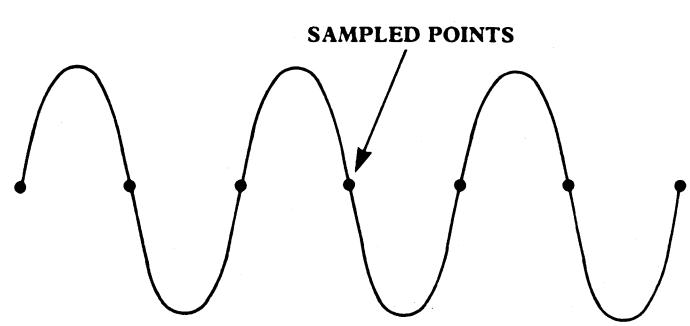

Figure 8 - A sine wave with frequency F, sampled at a rate 2F

Now consider the case where the sampling rate is four times the sine wave frequency, as in figure 9. In either of the situations illustrated in figure 9, although the signal data is somewhat skimpy, we can reconstruct the signal from the data. The algorithm used for reconstruction must include some assumptions about the shape of the signal, since the sampled data doesn't indicate whether it was a sine wave, a square wave, or a triangle wave, for instance (figure 10). Let's assume that we decide to reconstruct the signal as a sine wave. The effect on our uncertainty about the shape of the original signal is exactly the same as band-limiting the signal. Had we input a square wave, a sine wave, or a triangle wave into an analog oscilloscope with a frequency equal to or greater than the oscilloscope's -3 dB bandwidth, we would see a sine wave on the screen. This is an intuitive illustration of the similarity between the effects of sampling rate and bandwidth.

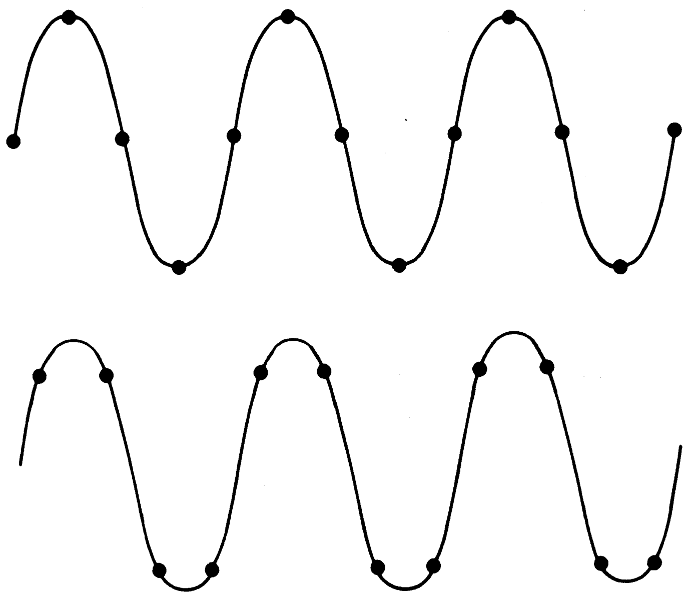

Figure 9 - A sine wave with frequency F, sampled at a rate 4F, shown in two sampling-phase possibilities. The samples contain sufficient data to recover the signal's correct amplitude and phase in either situation illustrated.

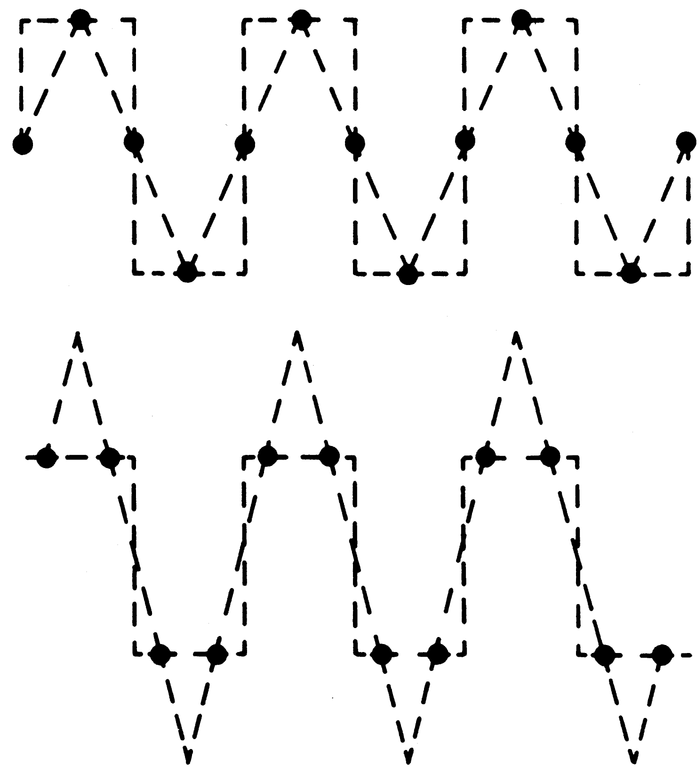

Figure 10 - Either set of samples in figure 9 might represent a square wave, a triangle wave, or any other arbitrary waveshape. The important point to note is that if we reconstruct a sine wave from the samples, we have as much information about the original signal as if we had passed the signal through a band-limiting filter that removed all frequencies but the fundamental. This is equivalent to viewing the signal with an oscilloscope with a bandwidth equal to F.

If the sampling rate is ten times the signal frequency, as in figure 11, an excellent reconstruction is possible by merely connecting the samples with straight vectors. With this sample density, the eye can interpret the original signal visually with negligible error.

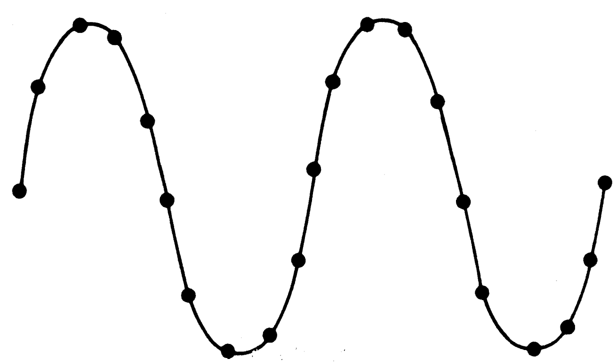

Figure 11 - A sine wave with frequency F sampled at a rate 10F; reconstruction can be accomplished by merely connecting the dots.

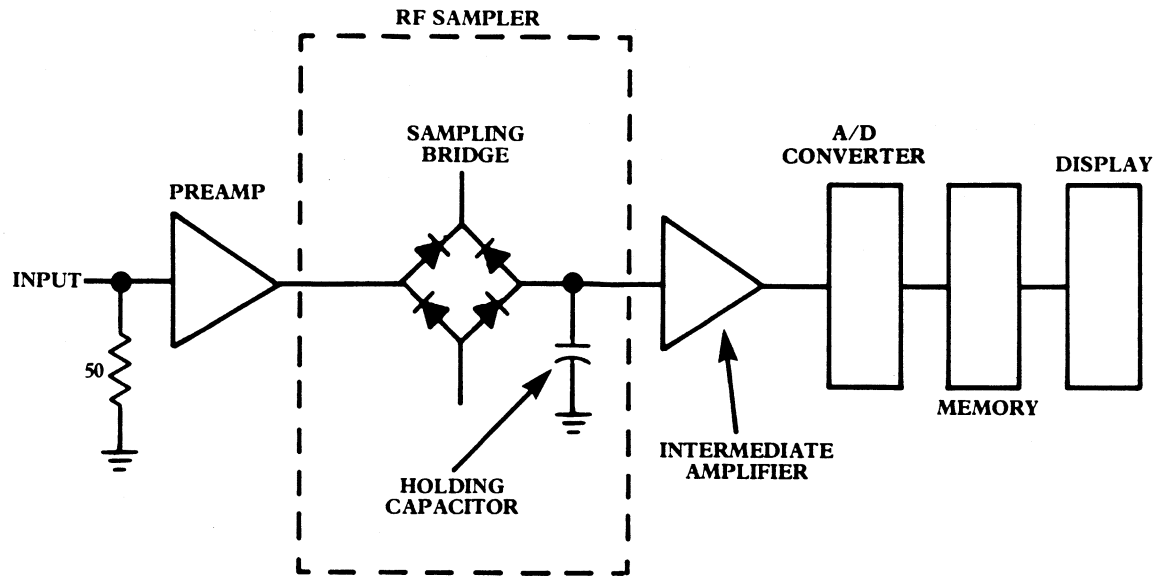

Figure 12 - HP 54100A/D vertical section block diagram.

As the signal frequency approaches the Nyquist limit, the assumptions in the reconstruction algorithm regarding the signal shape must become more dominant. Figure 8 shows that at the Nyquist limit, the assumptions inherent in the reconstruction algorithm dominate the reconstructed waveshape entirely, since the sampled signal data leaves the waveshape entirely up to the imagination. The effect is the same as a filter with a very high equivalent Q. The effect on the time domain and frequency domain response of the oscilloscope is the same as using extreme underdamping to extend the bandwidth of an analog oscilloscope. For this reason, few digitizing oscilloscopes or waveform analyzers offer reconstruction algorithms that work close to the Nyquist limit. For example, the HP 54200A/D has a 200 megasample/second digitizing rate. Its digital reconstruction filter is limited to a 50 MHz bandwidth, or one-fourth the sampling rate. This ensures minimum distortion in the reconstructed signal due to undersampling effects.

Some digitizing oscilloscopes have a bandwidth many times higher than their sampling rate. For example, the HP 54100A/D has a sampling rate of 40 MHz and a bandwidth of 1 GHz. This is achieved by using a wideband preamp and a narrow-aperture RF sampler ahead of the A/D converter (figure 12). To view a high-frequency signal, many samples acquired at different times are overlaid in correct time relationship to one another (figure 4). The time relationship is preserved by the trigger circuit. The bandwidth for repetitive signals is limited only by the bandwidth of the preamplifier and the aperture width of the sampler.

But what about the single-shot bandwidth of the HP 54100A/D? I will give an illustration to show that this is an incorrectly-phrased question that does not have a simple answer, and I will then show a simple way around the dilemma by re-phrasing the question to ask how the sampling rate affects the measurement.

If bandwidth is defined to be related to the amplitude error associated with the signal's rate-of-change, consider the case illustrated in figure 13. Here we have a single-shot, 1 ns wide pulse. In the case illustrated, the HP 54100A/D fortuitously acquired a sample located at the peak of the pulse. In random repetitive sampling, the sample has an equal probability of occurring anywhere. So, for a 1 ns wide pulse and a sampling rate of 40 megasamples/second, the probability of a sample coinciding with the pulse top is 2.5×10^{-2}. Because the input amplifier has a bandwidth of 1 GHz, and the RF sampler has a 350 ps aperture, this sample will be digitized at the full, correct amplitude. Thus, it could be argued that the single-shot bandwidth of the HP 54100A/D is 1 GHz.

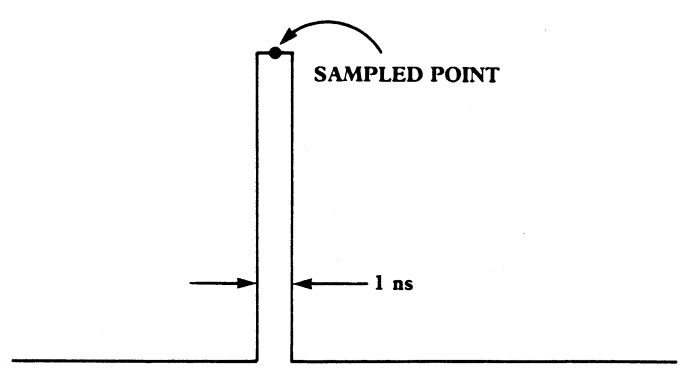

Figure 13 - The HP 54100A/D acquired a sample at the peak of a 1 ns wide pulse. The sample is at the correct amplitude.

The effect of the sampling rate in this case can again be described in terms of uncertainty. We don't know from examining the digitized record whether the pulse was 1 ns wide or 49 ns wide, or even if it was a pulse (figure 14). We also cannot determine the amplitude, since we don't know whether the sample coincided with a maximum on the signal. We can only distinguish one pulse from another in a single-shot acquisition for pulses > 50 ns wide. This is the same limit that would exist in an analog oscilloscope with a 20 MHz bandwidth.

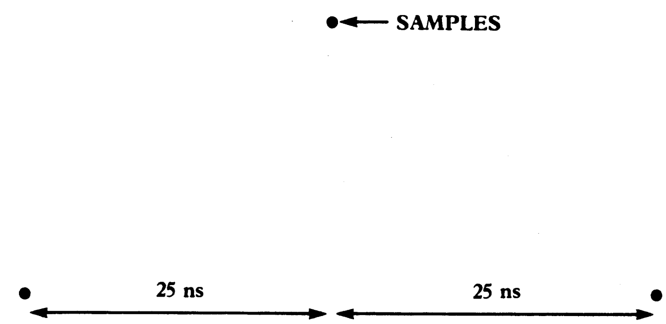

Figure 14 - Given only the samples acquired by the HP 54100A/D in a single acquisition on the signal shown in figure 13, our information about the signal is very limited.

This example shows why digitizing oscilloscopes that are optimized for single-shot measurements, or that operate only in the real-time sampling mode like the HP 54200A/D, often incorporate analog bandwidth limiting and/or digital filtering.

If the incoming signal had been band-limited to remove frequencies above 20 MHz, or the data were digitally filtered, then the ambiguity in figure 14 would not be present; the value of the sample acquired on the pulse peak would be quite small. Where digital filtering or analog band-limiting is not used, the oscilloscope user must use some judgment in interpreting the data in a single-shot measurement. The effective "bandwidth" of a single-shot measurement in terms of useful data about the signal has an absolute limit at the Nyquist rate. In the absence of digital filtering and reconstruction, a more conservative approach is to assume that any frequency components above one-tenth the sampling rate may be aliased, since you must depend on the eye for reconstructing the signal (refer to figures 8, 9, and 11). This would imply that you should be cautious about the results of single-shot measurements above 4 MHz using the HP 54100A/D, although there is some information present about the input signal up to 20 MHz.

The Bottom Line

The bottom line is that we really don't care what the relationship is between sampling rate and bandwidth; what we care about is the effect each has on the measurement. Since there is not a simple, unambiguous formula to relate sampling rate to bandwidth, it is far better and easier to analyze the effect of sampling rate on the measurement directly rather than indirectly through relating sampling rate to bandwidth. In addition to the familiar time and frequency domains, we can use the probability domain to advantage in analyzing digitizing oscilloscopes relative to an application requirement and comparing them to one another. The references include guides to understanding the relationships between the probability domain and the more familiar time and frequency domains in as much mathematical detail as the reader desires.

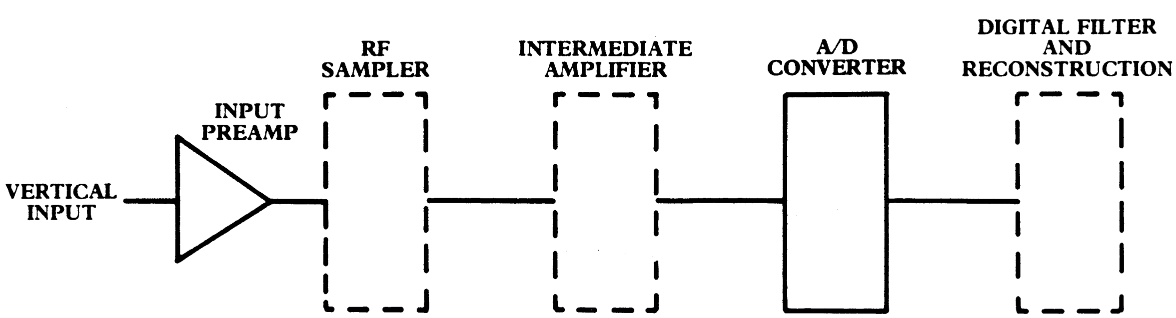

Figure 15 - Digitizing oscilloscope block diagram

Selecting The Right Oscilloscope For The Application

The first question to ask when selecting an oscilloscope, analog or digitizing, is "what is the measurement requirement?" To select the right digitizing oscilloscope for a given application, it is necessary to look at three items: the repetitive bandwidth, the single-shot bandwidth (if different), and the sampling rate. Each of these must be evaluated in light of its effect on time-interval or amplitude measurement error or uncertainty. The elements of a digitizing oscilloscope that affect these variables are shown in the block diagram (figure 15). Not every digitizing oscilloscope incorporates all of these elements; dashed-line blocks are optional items.

These elements affect the performance parameters in the following ways:

CIRCUIT BLOCK	AFFECTS
Input preamp	Repetitive bandwidth Single-shot bandwidth
RF sampler	Repetitive bandwidth Sample rate
Intermediate amplifier	Single-shot bandwidth
A/D converter	* Repetitive bandwidth Single-shot bandwidth Sample rate
Digital filter and reconstruction	Single-shot bandwidth Repetitive bandwidth

* Only if there is no RF sampler ahead of the A/D converter.

As an example of this process, the following selection tree compares the HP 54100A/D and HP 54200A/D digitizing oscilloscopes from the viewpoint of the application requirement.

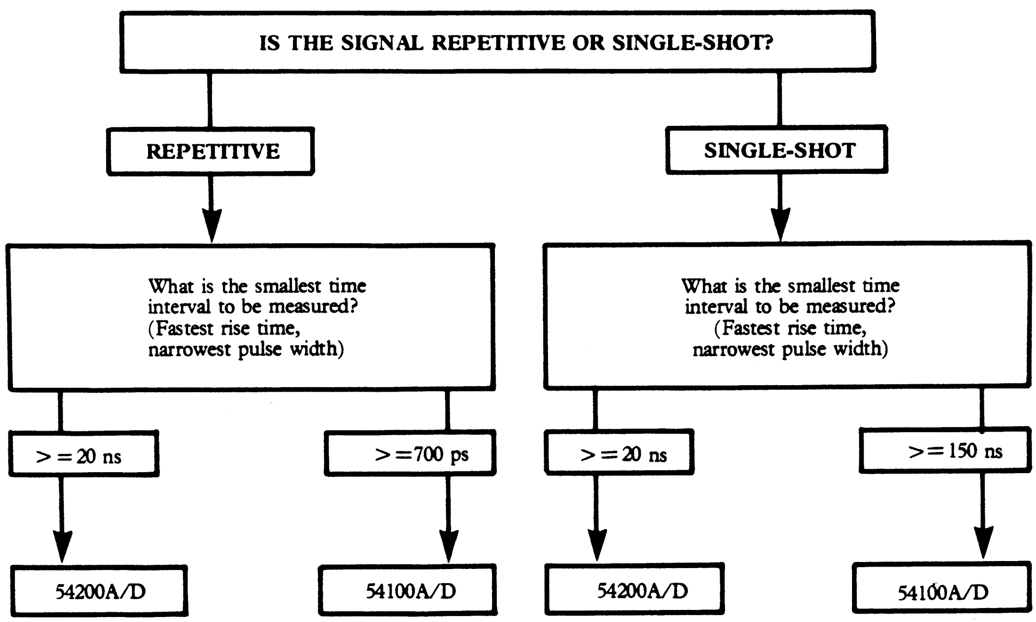

Figure 16 - Oscilloscope selection tree, based on time- interval measurement requirements.

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